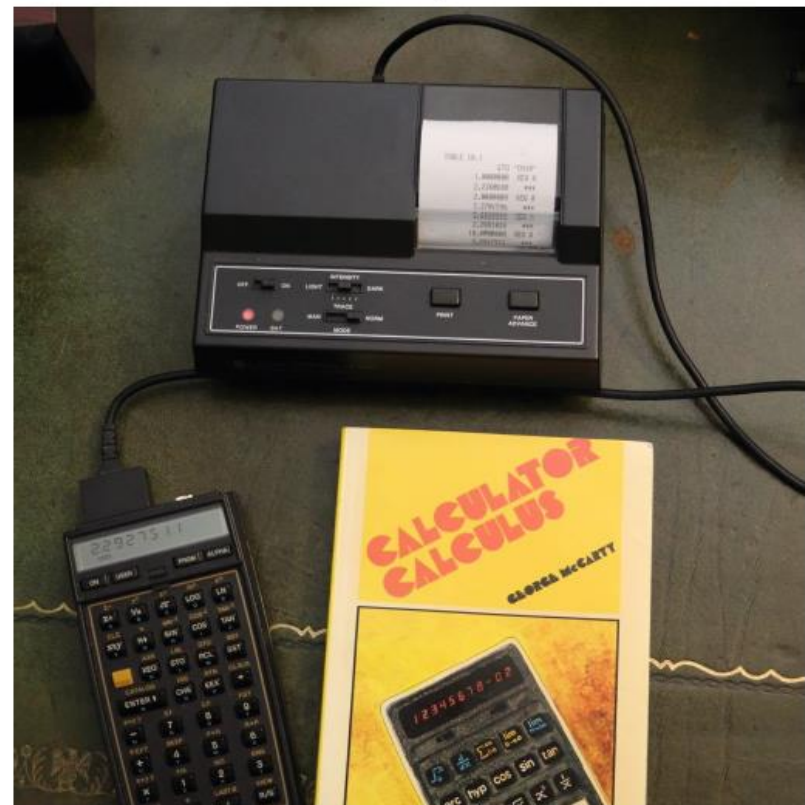


Creating
Calculator Calculus
Companion
140 pages of action
packed RPN calculus
programs

Dave Hayden
HHC 2025

Calculator Calculus Companion

By
David Hayden



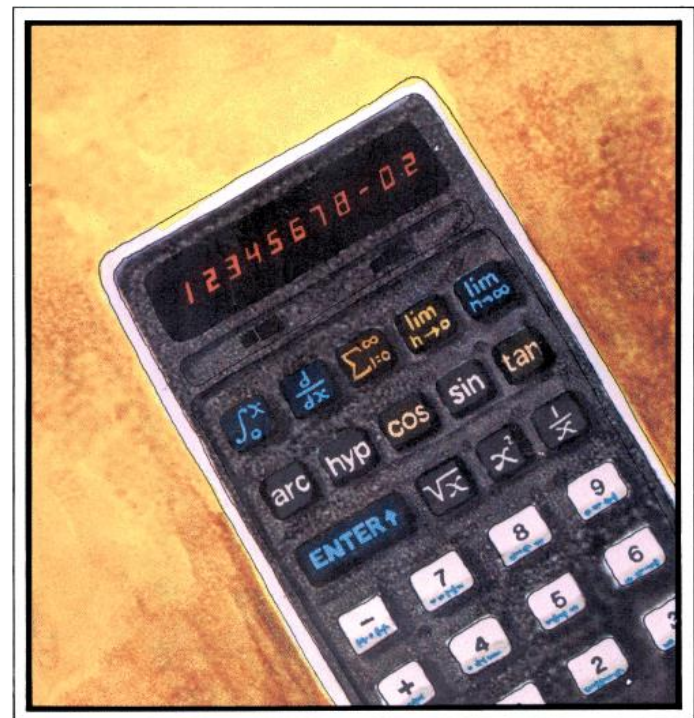
Programs to compute the examples in
George McCarty's *Calculator Calculus*

What is *Calculator Calculus*?

“This book is about the calculus. ... [I]t uses the pocket calculator to illustrate the theory.” - from the *Calculator Calculus* preface.

- First published in 1975
- PDF:
<https://literature.hpcalc.org/items/1537>

Calculator Calculus



George McCarty

What's in it?

- Calculus Stuff
- Each Chapter has Examples, Exercises and Problems.

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EXAMPLE: $\sqrt{67.89}$

Suppose we want to compute $\sqrt{67.89}$. We can first make a rough guess at the answer of 8, since $8^2 = 64$, which is fairly close to 67.89. Now we use an arithmetic trick to improve our guess. We let y stand for the number we want to find, so that $y^2 = 67.89$. Then we write

$$\begin{aligned}y^2 &= 67.89 \\y^2 + 8y &= 67.89 - 64 + 8y + 64 \\y(y+8) &= 3.89 + 8(y+8) \\y &= \frac{3.89}{y+8} + 8.\end{aligned}$$

This equation will be satisfied by y and by no other number. Since we wish to improve our guess for $\sqrt{67.89}$, we will experiment by

2

regarding the *two appearances* of the number y in this equation as *two different numbers* that are related by the equation. That is, we relabel our guess g as $g = y_0$ and calculate

$$y_1 = \frac{3.89}{y_0+8} + 8.$$

TABLE 1.1

y_0	=	8.
y_1	=	8.2431250
y_2	=	8.2394859
y_3	=	8.2395396
y_4	=	8.2395388
y_5	=	8.2395388

Looks like an
opportunity
for a program!

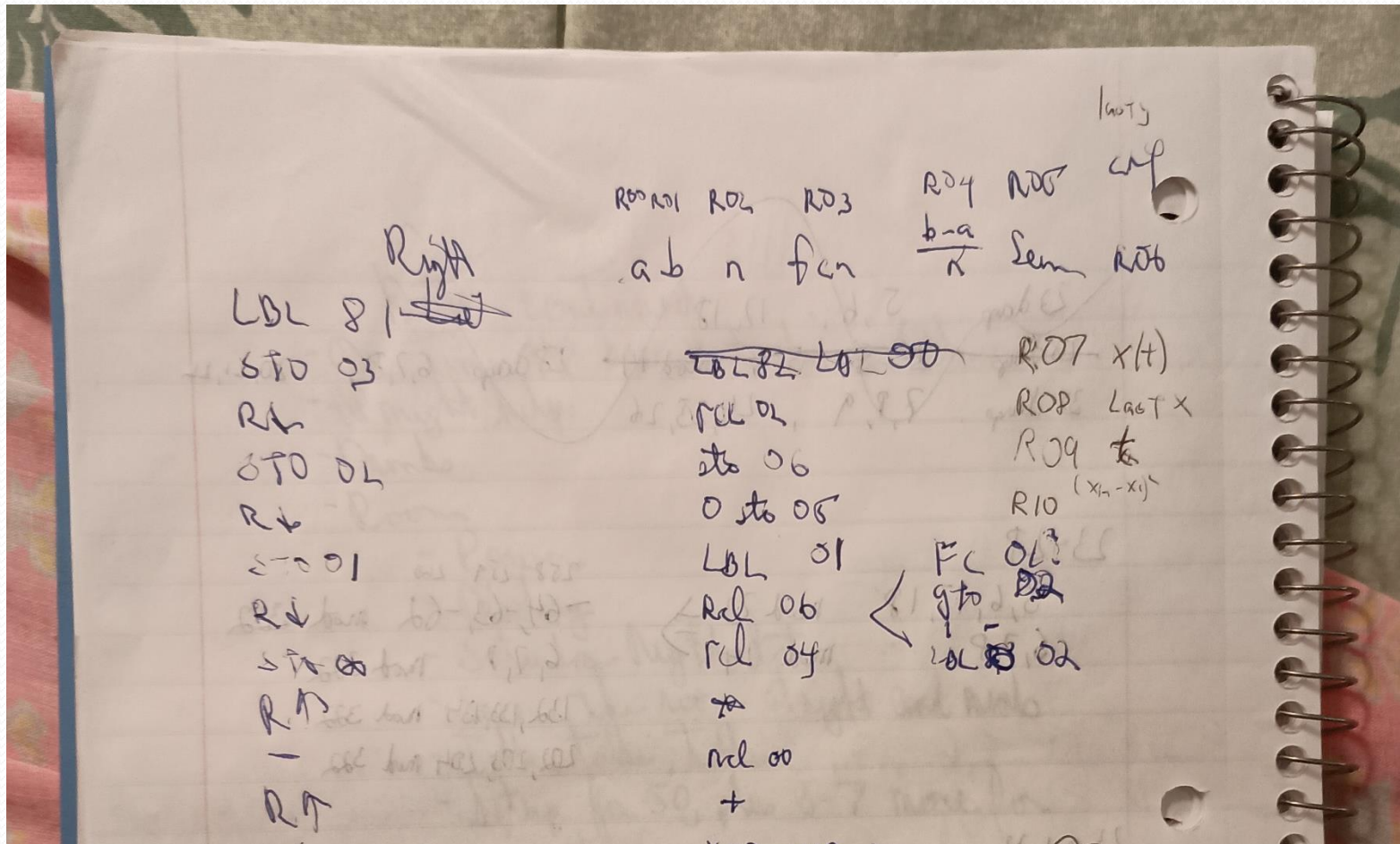
Origins of Calculator Calculus Companion

- Got letter-size *Calculator Calculus* (from the Prize Table?) at Some Point.
- Donated it back to Prize Table a Year or Two Later?
- Bought a half-letter sized copy in 2019.
- Nice examples!
- Wrote programs before bedtime 20 minutes at a time on a DM42 just for fun.

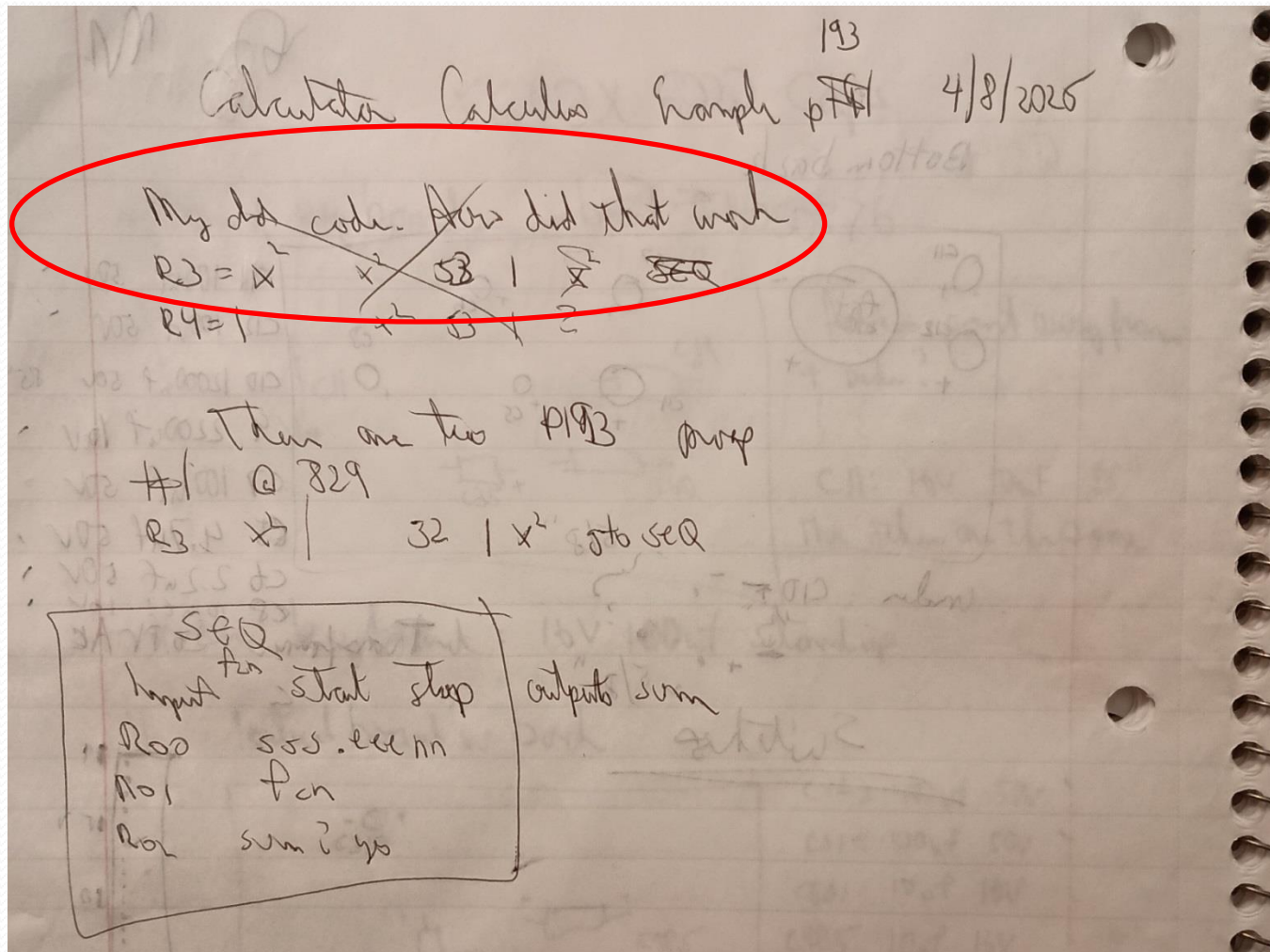
First Draft of Programs

- Decided to write programs for all examples.
 - Well ... all non-trivial examples
- Wrote it as one program on DM42.
- Printed it out. I threw away that annotated printout just a few days ago!
- Retest the programs. Wait. How does that one work? What does this do? This code sucks! Ugh.

My Notebook



“My old code. How did that work”



Cleaning up the Code

- One large program would barely fit in a 41CX.
- Divided it into one per chapter.
- One “utilities” program for all common subroutines.
 - Individual Utility programs would be inconvenient.
 - Also, the Utility programs use utility subroutines....
 - “Load program ‘CHo3’, and Utility programs A, D, and F”.

Birth of the Book

- Started writing some simple documentation.
- “Hmm. If I want to give this to anyone else, the documentation should be better.”
- “Hmm ... and it should contains listings...”
- “... and bar code would be cool.”

Thanks to Joe Horn!

In an email, or conversation, or thread, I mentioned the idea of this book and he casually replied that he thought it was a good idea.

Organization

After some fits and starts, settled on:

- Chapter N
 - Page NN: <title of original example>
Description
 - Instructions
 - Example
 - ...
 - Program Listing
- Appendix
 - Bar codes

$f(x) = \sqrt{1-x^2}$ in the first quadrant (Figure 7.2). Suppose we first divide the interval $[0,1]$ into four pieces $[0, 1/4]$, $[1/4, 1/2]$, $[1/2, 3/4]$, and $[3/4, 1]$. Above each piece we construct the largest rectangle that has that piece for a base and lies inside the region we are measuring. Each rectangle has width $1/4$ and height $f(x)$ for x at the right hand edge (Figure 7.3). The area of these pieces is, respectively,

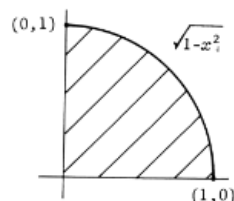


Figure 7.2

$$\frac{\sqrt{1-1/16}}{4}, \frac{\sqrt{1-1/4}}{4}, \frac{\sqrt{1-9/16}}{4},$$

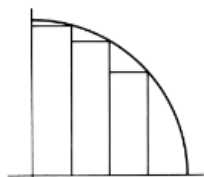


Figure 7.3

and 0, and the sum of these areas is $2.4957091/4$. This is a very poor approximation to $\pi/4$.

Next, we construct over the same pieces of the interval $[0,1]$ the smallest rectangles on those pieces as bases that together contain the region in question (Figure 7.4). Their areas are, respectively,

$$\frac{1}{4}, \frac{\sqrt{1-1/16}}{4}, \frac{\sqrt{1-1/4}}{4}, \frac{\sqrt{1-9/16}}{4},$$

and the sum of these areas is $3.4957091/4$, which is the earlier

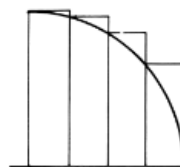


Figure 7.4

Chapter 7 – Definite Integrals

Chapter 7 – Definite Integrals

Page 82: π and the Area of a Disc

Subroutines A and B estimate $\int_0^1 \sqrt{1-x^2}$ by computing 4 times the area of the four rectangles in Figures 7.3 and 7.4 on page 83 in *Calculator Calculus* respectively.

Subroutines C and D are similar to A and B, except (1) they divide the area into 10 rectangles instead of four as show in Figures 7.5 and 7.6 on pages 85 and 86 in *Calculator Calculus* respectively, and (2) they don't multiply the resulting sum by 4.

Instructions

1. Load program Chapter 7 from the bar code in the appendix or the listing at the end of this chapter.
2. Load the Utilities program.
3. GTO "CH07" to activate the local labels.
4. XEQ A, B, C, or D to get the values shown on pages 83–84.

Example 1

Compute the values shown next to Figures 7.3 and 7.4 on page 83 in *Calculator Calculus*

Keystrokes	Display	Comments
FIX 7		
XEQ A	2.4957091	This is the value shown next to Figure 7.3.
XEQ B	3.4957091	This is the value shown next to Figure 7.4.

Book Format

- Voyager format because I had it from the 15C-LE documents and 10C owner's manual for Scottie.
- Nice “Examples” table format.
- Kept Instructions as regular as possible:
 1. Load program Chapter 11 from the bar code in the appendix or the listing at the end of this chapter.
 2. Load the Utilities program.
 3. GTO “CH11” to activate local labels.
 4. Enter n .
 5. XEQ G to compute the sum.
- Text Font is Palatino Linotype 11pt which looks somewhat like the Voyager manuals.
- Disambiguate page # references
 - *Calculator Calculus* or *Calculator Calculus Companion*?

By about Chapter 5, I settled on a format:

Heading 1: Chapter n – Text from Calculator Calculus

Heading 2: Page n: Text from Calculator Calculus Example

Heading 3: Instructions, Example,

Program Listing

Example Table:

Keystrokes Display
Comments

Chapter 1 – Squares, Square Roots and the Quadratic Formula

Page 2: $\sqrt{67.89}$

Subroutine A estimates $\sqrt{67.89}$ from an initial estimate y_0 by computing $y_{i+1} = \frac{3.89}{y_i+8} + 8$ as described near the bottom of page 3 in *Calculator Calculus*.

Instructions

1. Load program Chapter 1 from the bar code in the appendix or the listing at the end of this chapter.
2. GTO "CH01" to activate the local labels.
3. Enter an initial guess y_0 .
4. XEQ A to compute y_1 .
5. Press R/S to compute subsequent values of y_i .

Example:

Compute successive estimates of $\sqrt{67.89}$ from an initial guess of 8 as shown on page 3 and culminating in Table 1.1 on page 4 in *Calculator Calculus*.

Keystrokes	Display	Comments
FIX 7		
8 XEQ A	8 . 2431250	First approximation y_1 .
R/S	8 . 2394859	y_2
R/S	8 . 2395396	y_3
R/S	8 . 2395388	y_4

Good Headings make a good Table of Contents

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What's up here?

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??

Maybe a Bit Too Good....

Chapter 5 – Maxima, Minima, and the Mean value Theorem

There are no examples that require programs in this chapter.

If I left out Chapter 5, someone would say “hey, you skipped a chapter.”

The Program Listings

- Each Program starts with “LBL CH*nn*” for CAT 1
- Highlighted labels.
- GTO, XEQ show target line number.
- Lots of comments because I hate undocumented code.
- Decided to put them at the end of each chapter instead of each example.

Program Listing Example

Chapter 2 – More Functions and Graphs

Instructions	Comments
01 LBL "CH02"	
02 LBL A	Input x_i
03 X^2	
04 3	
05 -	$x_i^2 - 3$
06 1/X	$x_{i+1} = 1 / (x_i^2 - 3)$
07 RTN	
08 LBL B	Input x . Output $x^3 - 3x - 1$
09 ENTER	
10 X^2	
11 3	
12 -	$x^2 - 3$
13 *	$x^3 - 3x$
14 1	
15 -	$x^3 - 3x - 1$
16 RTN	
17 LBL C	Input x_i Output $\text{SQRT}(1/x_i + 3)$

Utilities Program

- Frequently Used Subroutines
- RSUM, LSUM
 - Estimate integral of $f(x)$ from a to b using n rectangles whose height is $f(x)$ at the right or left side.
- TSUM
 - Like RSUM and LSUM but uses trapezoids.
- STOARGS
 - Store a , b , n , and f arguments for RSUM, LSUM, and TSUM.
- SERIES
 - Compute $\sum_{x=a}^b f(x)$.
- LIM
 - Show (and print) $f(x)$ for values near limit L .

Utilities Program: Recall This Example

Chapter 7 – Definite Integrals

$f(x) = \sqrt{1-x^2}$ in the first quadrant (Figure 7.2). Suppose we first divide the interval $[0,1]$ into four pieces $[0, 1/4]$, $[1/4, 1/2]$, $[1/2, 3/4]$, and $[3/4, 1]$. Above each piece we construct the largest rectangle that has that piece for a base and lies inside the region we are measuring. Each rectangle has width $1/4$ and height $f(x)$ for x at the right hand edge (Figure 7.3). The area of these pieces is, respectively,

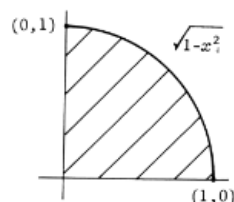


Figure 7.2

$$\frac{\sqrt{1-1/16}}{4}, \frac{\sqrt{1-1/4}}{4}, \frac{\sqrt{1-9/16}}{4},$$

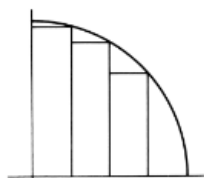


Figure 7.3

and 0, and the sum of these areas is $2.4957091/4$. This is a very poor approximation to $\pi/4$.

Next, we construct over the same pieces of the interval $[0,1]$ the smallest rectangles on those pieces as bases that together contain the region in question (Figure 7.4). Their areas are, respectively,

$$\frac{1}{4}, \frac{\sqrt{1-1/16}}{4}, \frac{\sqrt{1-1/4}}{4}, \frac{\sqrt{1-9/16}}{4},$$

and the sum of these areas is $3.4957091/4$, which is the earlier

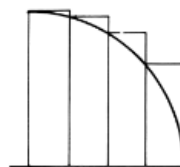


Figure 7.4

Chapter 7 – Definite Integrals

Page 82: π and the Area of a Disc

Subroutines A and B estimate $\int_0^1 \sqrt{1-x^2}$ by computing 4 times the area of the four rectangles in Figures 7.3 and 7.4 on page 83 in *Calculator Calculus* respectively.

Subroutines C and D are similar to A and B, except (1) they divide the area into 10 rectangles instead of four as show in Figures 7.5 and 7.6 on pages 85 and 86 in *Calculator Calculus* respectively, and (2) they don't multiply the resulting sum by 4.

Instructions

1. Load program Chapter 7 from the bar code in the appendix or the listing at the end of this chapter.
2. Load the Utilities program.
3. GTO "CH07" to activate the local labels.
4. XEQ A, B, C, or D to get the values shown on pages 83–84.

Example 1

Compute the values shown next to Figures 7.3 and 7.4 on page 83 in *Calculator Calculus*

Keystrokes	Display	Comments
FIX 7		
XEQ A	2.4957091	This is the value shown next to Figure 7.3.
XEQ B	3.4957091	This is the value shown next to Figure 7.4.

Program Listing Example with Utilities

Program Listing

Subroutines A, B, C, and D use subroutines "RSUM" and "LSUM" from the Utilities program to do the heavy lifting. See "RSUM" on page 96 and "LSUM" on page 97.

Instructions	Comments
01 LBL "CH07"	
02 LBL "F83"	Input x . Output $\sqrt{1-x^2}$
03 X^2	
04 1	

36

Chapter 1

05 $X \leftrightarrow Y$	
06 -	$1 - x^2$
07 SQRT	
08 RTN	

09 LBL A	
10 0	a , the lower bound of integration
11 1	b , the upper bound of integration
12 4	n , the number of rectangles to divide the area from a to b into
13 "F83"	f , the alpha label of the subroutine that computes $f(x)$, the function being integrated.
14 XEQ "RSUM"	(Utilities program line 17) Estimate the integral of $f(x)$ from a to b by dividing it into n rectangles whose height is $f(x)$ on the right side of the rectangle.
15 4	
16 *	Multiply by 4 to get the area of the full circle
17 RTN	

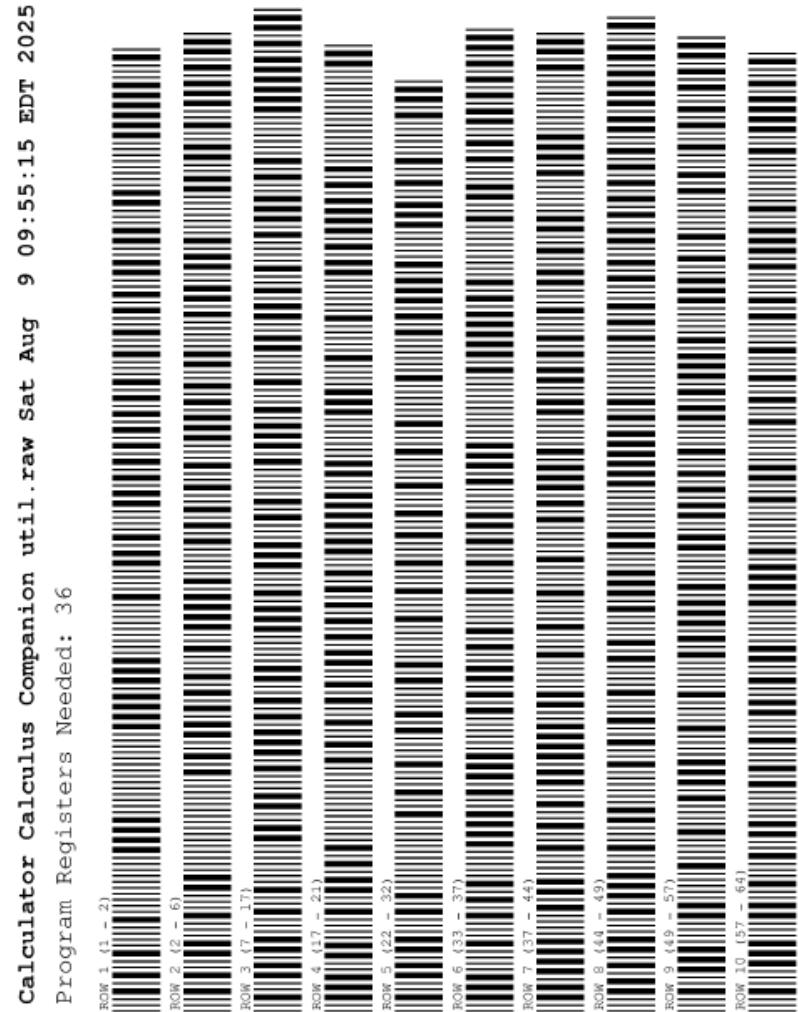
Document Size

- First version was Letter Size (~A4).
- Half Letter size (~A5) was only a few page longer.
- My printer doesn't have a half-letter setting and won't print double sided with custom sizes.
- Document is A5.
- Printed it on half-letter paper I cut myself.

Bar Code

- Used hp41uc to create PCL
- Online PCL-to-GIF converter.
- ThumbsPlus to crop files to fit the page

Appendix 1 – 41C Bar Code



An Error in *Calculator Calculus*?

harmonic series,

$$\sum_1 \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

We have calculated S_{10} to be 2.9289683, yet S_{100} is only 5.1774272.

- Page 147 in *Calculator Calculus*
- I got 5.1873775
- Off by about $1/(100 + 1/2)$. Maybe they computed S_{99} ?
- Maybe someone can figure it out.

Editing

- Hyphen vs. minus sign, vs. en-dash, vs. em-dash
- Italics in equations and “*Calculator Calculus*”
- Cropping bar codes
- One Error in Example Output: DM₄₂ vs. 41C.

Binding

- Printed manuals are nice.
- Saw a Fastback strip binding machine on facebook marketplace.
- Thank you Scottie, who bought it for me for my birthday.
- Needed some maintenance.
- Bought a large paper cutter.
- Bought heavy card stock for covers
- See Richard Schwartz's HHC 2014 Presentation on Book Binding. The method with Aleene's Fabric Glue makes great lay-flat documents.

Fastback 15x Strip Binder



HardwareFactoryStore Finger Paper Cutter

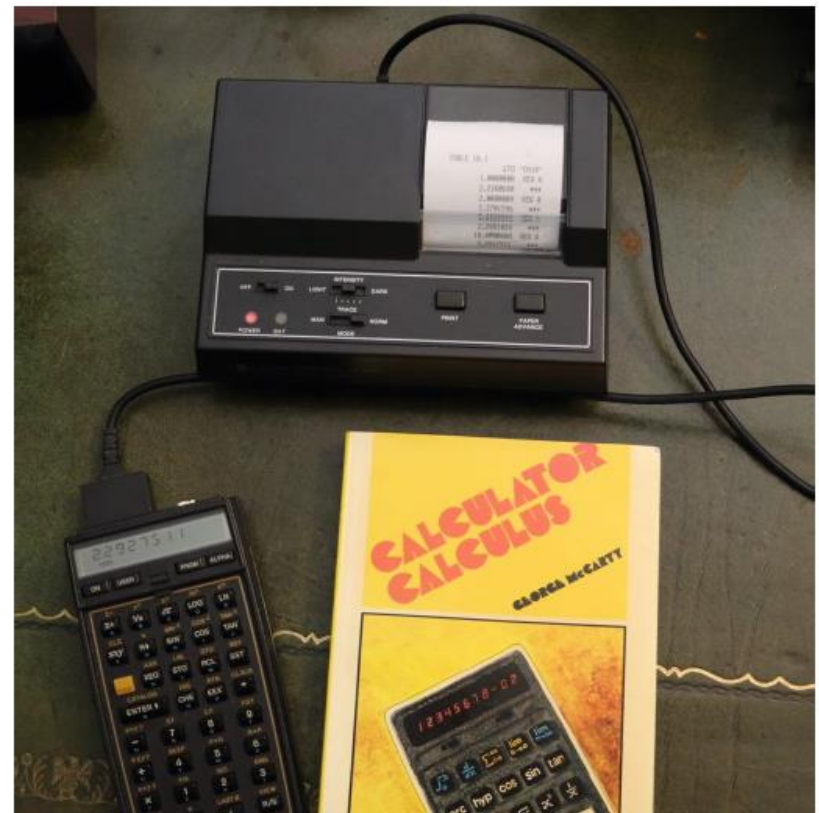


Cover Photo

Calculator Calculus Companion

By
David Hayden

- My 41CL
- Printout shows creating Table 10.1



Programs to compute the examples in
George McCarty's *Calculator Calculus*

Getting the Book

- Eric's literature.hpcalc.org has pdfs of *Calculator Calculus* and *Calculator Calculus Companion*.
- <https://www.hpmuseum.org/forum/thread-23998.html> has the code (.raw files) and a link to Eric's site.
- Printed copies here at HHC 2025?
- Let me know and I'll print a copy for you.

The Future

- Print 17,000 copies? Maybe not.
- More Corrections seem inevitable.
- Module (.mod) file to store all the programs?
- Do the Exercises and Problems?